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COMPLEX MASSES AND ACAUSAL PROPAGATION IN FIELD THEORY *

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Abstract: It is shown that in formulating a field theory with complex masses, keeping much closer to a detailed space-time description than existing treatments, one is naturally led to a modified Yang-Feldman equation.

A consistent interpretation emerges, in which the complex masses are not associated with particles, but with a modification of the field propagation law, which becomes acausal.

The indefinite metric of the usual treatments plays no role here.

An investigation of both the case of interaction with an external potential, and of the fully quantized theory is carried out.

No unitarity, or Lorentz invariance troubles arise, but in the case of a fully quantized theory, the contribution of virtual states with arbitrarily high energy, leads to an enhancement of the basic acausality of the propagation law, in a way that may lead to macroscopic deviations from causality.

1. INTRODUCTION

Considerable interest has been given recently [1–4] to theories with complex regulator masses. (For an earlier related discussion see ref. [5].) Through a set of modified Feynman rules [1], supplemented by specific integration prescriptions [2, 3] a perturbative expansion of an unitary, Lorentz invariant and finite S-matrix has been obtained.

In those treatments, field theory plays basically only the role of an heuristic guide to the derivation of the modified Feyman rules and the connection between the final S-matrix and a specific field theory, with given equations of motion remains unclear.

In this paper we shall follow a path much closer to usual field theoretical concepts, making only the minimal modifications necessary to consistently incorporate complex masses into the theory.

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To see the essentials of the problem we shall introduce complex masses in the simplest way i.e. through a higher order differential equation with masses m (real) and M, M^*,

$$\frac{(\square + m^2)(\square + M^2)(\square + M^{*2})}{|M^2 - m^2|^2} \phi(x) = J(x) , \tag{1}$$

where $J(x)$ is an as yet unspecified source and the normalization factor $|M^2 - m^2|^{-2}$ is chosen so that formally the equation goes into a Klein-Gordon one for $M \to \infty$.

It is of course possible to obtain (1) from a Lagrangian and associate to it a (pseudo) canonical formalism [6], but this will be of little use in our discussion.

We shall employ in solving (1) the Yang-Feldman method [7] which conventionally would lead to:

$$\phi(x) = \phi^{\text{in}}(x) + \int G_{\text{R}}(x - y) J(y) \, \mathrm{d}^4 y , \tag{2}$$

with the retarded function given by

$$G_{\text{R}}(x) = -\frac{1}{(2\pi)^4} \int_{C_{\text{R}}} \frac{\mathrm{d}^4 k \, \mathrm{e}^{-ikx} |M^2 - m^2|^2}{(k^2 - m^2)(k^2 - M^2)(k^2 - M^{*2})} , \tag{3}$$

and the retarded contour shown in fig. 1.

In (2) ϕ^{in} corresponds to the solution of the free equation

$$\phi^{\text{in}}(x) = \phi_m^{\text{in}}(x) + \sqrt{\frac{M^{*2} - m^2}{M^2 - M^{*2}}} \, \phi_M^{\text{in}}(x) + \sqrt{\frac{M^2 - m^2}{M^{*2} - M^2}} \, \phi_{M^*}^{\text{in}}(x) , \tag{4}$$

where

$$\phi_m^{\text{in}}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{\mathrm{d}^3 k}{\sqrt{2}^{\,4} (k^2 + m^2)^{\frac{1}{4}}} \{a_{\text{in}}(k) \, \mathrm{e}^{-ikx} + a_{\text{in}}^+(k) \, \mathrm{e}^{ikx}\} , \tag{5}$$

$$\phi_M^{\text{in}}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{\mathrm{d}^3 k}{\sqrt{2}^{\,4} (k^2 + M^2)^{\frac{1}{4}}} \{c_{\text{in}}(k) \, \mathrm{e}^{-ikx} + b_{\text{in}}^+(k) \, \mathrm{e}^{ikx}\} ,$$

$$\phi_{M^*}^{\text{in}}(x) = (\phi_M^{\text{in}}(x))^+ . \tag{6}$$

In (5) and (6)

$$[a_{\text{in}}(k) , a_{\text{in}}^+(k')] = \delta(k - k') , \tag{7}$$

$$[c_{\text{in}}(k)\,,\,b^{+}_{\text{in}}(k')] = [b_{\text{in}}(k)\,,\,c^{+}_{\text{in}}(k')] = \delta\,(k-k')\,, \tag{8}$$

all other commutators vanishing. These commutation relations, together with the interpretation of $c(k)$, $b(k)$, as annihilation operators, imply, of course, an indefinite metric in the state space.

One could try now to set up a perturbation theory based on (2). However, both the Green function and the incoming fields, have a time dependence that blows up exponentially, leading to meaningless integrals.

This is an old story. It was the same difficulty that led Lee and Wick [1] to their modified Feynman rules for the S-matrix. Our insistence in working with the field operators rather than only with the S-matrix will lead us, however, to different rules from the Lee-Wick ones.

In our opinion, a field theoretical framework, with a much more detailed space-time description than a pure S-matrix theory, is of considerable interest in clarifying a number of questions related to the introduction of complex masses. In particular, the violation of causality can be analysed in a direct way through the response of the system to an external classical perturbation.

In order to understand the troubles with eq. (2), as well as to have a clue to its necessary modifications, we start in sect. 2 with the particularly simple example of interactions with an external classical potential, and proceed to a discussion of the fully quantized theory in sect. 3.

2. EXTERNAL POTENTIAL

We take here $J(x) = V(x)\,\phi\,(x)$ in (1). In this case, it is clear that, as long as the potential has finite-duration, there are no convergence problems, and one can show the existence of a pseudo-unitary S-matrix adapting the methods used in [8]. Physical positive definite unitarity is however, violated, due to transitions between real and complex energy states. In the Born approximation, for instance, one has, as usual, the transition amplitudes proportional to the Fourier transform of the potential. Complex energy transfers are possible, since we have the Fourier transform of a function of compact support in time, and therefore, an entire function of the energy. As the support of the potential in time, goes to infinity, those amplitudes describing complex energy transfers become divergent.In second order, even the amplitudes for "physical", real mass processes diverge, due to the contribution of intermediate states with complex energy. This divergence of "physical" amplitudes, should come as no surprise, considering that we have a pseudo-unitary theory, where cancellation between infinite positive and negative "probabilities" can occur.

It is also clear, that the naive argument of energy conservation, which for a static potential, would exclude transitions between real and complex energies, is incorrect.

In order to be able to have a meaningful theory based on eq. (1), it is unavoidable that we will have to modify (2) in such a way that no exponential blow-up in time occurs. It will turn out that in doing so, we will automatically enforce physical unitarity and obtain an unambiguous set of modified Feynman rules, different from the Lee-Wick [1] ones.

First, it is clear that, to avoid the exponential increase of the Green function it is necessary to introduce a new integration contour (along the real axis), so that

$$G_{\mathrm{R}m}(x) = -\frac{1}{(2\pi)^4}\int_{C_{\mathrm{R}m}} \frac{\mathrm{d}^4k\,\mathrm{e}^{-\mathrm{i}kx}|M^2-m^2|^2}{(k^2-m^2)(k^2-M^2)(k^2-M^{*2})} \tag{9}$$

with the modified contour $C_{\mathrm{R}m}$ given by fig. 1.

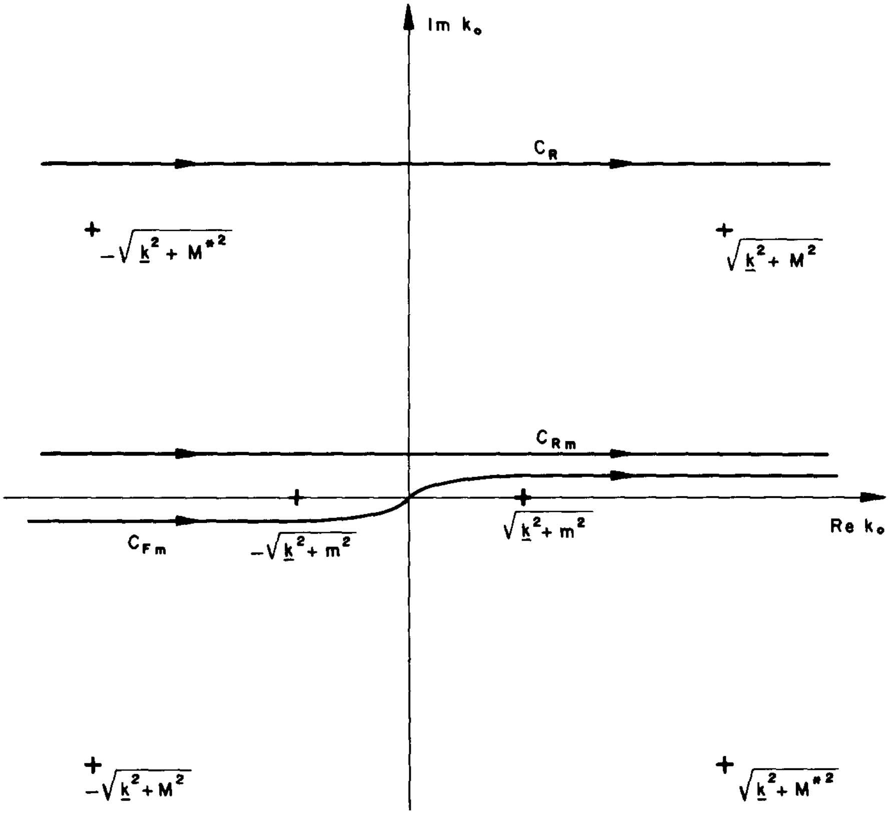

Fig. 1. The contours C_{R}, $C_{\mathrm{R}m}$ and $C_{\mathrm{F}m}$ for the Green function.

The new Green function G_{Rm} is no longer strictly retarded since the poles in the upper half plane will introduce a certain amount of acausality of the order of $1/\mathrm{Im}\,M$ [9].

That an acausality should appear in a theory with complex masses, would be expected in any case [1], and it is advantageous to have it explicit as early as possible. It should be stressed that this acausality is of a "primitive kind" [10], but will reflect itself in a violation of the relativistic Einstein causality (local commutativity).

Besides modifying one's Green function, one has also to eliminate the exponential growth from the incoming fields (4) themselves, and this will be simply done by taking as incoming configuration, just the field $\phi_m^{\mathrm{in}}(x)$. Physically this is very reasonable since we do not want the complex mass "particles" to have any asymptotic manifestation.

We arrive thus, at a modified Yang-Feldman equation

$$\phi(x) = \phi_m^{\mathrm{in}}(x) + \int G_{Rm}(x-y)\, J(y)\, \mathrm{d}^4 y \tag{10}$$

with $J(y) = V(y)\,\phi(y)$ for the case under consideration here. The solution of the integral eq. (10) is of course again a solution of the basic differential eq. (1) corresponding however to different boundary conditions.

It is clear that, since G_{Rm} is damped exponentially for $x_0 - y_0 < 0$ we have

$$\underset{x_0 \to -\infty}{\phi(x)} = \underset{x_0 \to -\infty}{\phi_m^{\mathrm{in}}(x)} \tag{11}$$

in the sense of L.S.Z. [11] and

$$\underset{x_0 \to \infty}{\phi(x)} \equiv \underset{x_0 \to \infty}{\phi_m^{\mathrm{out}}(x)}\,, \qquad (\Box + m^2)\,\phi_m^{\mathrm{out}}(x) = 0\,, \tag{12}$$

since only the oscillating part of G_{Rm} corresponding to the real mass m will survive for $x_0 \to \infty$.

It can be readily seen that the perturbation theory based on (10) is now free of divergences to every order in perturbation theory, even for static potentials.

We shall now verify that this theory leads to an unitary S matrix in every order. Writing the expansion of (10)

$$\phi(x) = \phi_m^{\mathrm{in}}(x) + \int G_{Rm}(x-y)\, V(y)\, \phi_m^{\mathrm{in}}(y)\, \mathrm{d}^4 y + \ldots \tag{13a}$$

Symbolically as

$$\phi = \frac{1}{1 - G_{Rm} V}\, \phi_m^{\mathrm{in}} \tag{13b}$$

we have

$$[\phi(x), \phi(y)] = \frac{1}{1 - G_{Rm}V} \frac{\Delta}{i} \frac{1}{1 - VG_{Am}} \tag{14}$$

with

$$G_{Am}(x) = G_{Rm}(-x), \tag{15}$$

$$\Delta(x-y) = i\,[\phi_m^{in}(x), \phi_m^{in}(y)] = G_{Rm}(x-y) - G_{Am}(x-y). \tag{16}$$

Using (16) one can write (14) as

$$[\phi(x), \phi(y)] = \frac{1}{i} \frac{1}{1 - G_{Rm}V} \frac{1}{V} - \frac{1}{V} \frac{1}{1 - VG_{Am}}, \tag{17}$$

leading to

$$[\phi(x), \phi(y)] = [\phi_m^{in}(x), \phi_m^{in}(y)] + \frac{1}{i}\Big(G_{Rm}VG_{Rm} + G_{Rm}VG_{Rm}VG_{Rm} + \ldots - G_{Am}VG_{Am} - G_{Am}VG_{Am}VG_{Am} - \ldots\Big), \tag{18}$$

with a clear symbolic meaning.

If G_{Rm} were a normal retarded propagator, eq. (18) would imply local commutativity for ϕ, and for x_0, y_0 larger than the time the potential is switched off,

$$[\phi(x), \phi(y)] = [\phi_m^{out}(x), \phi_m^{out}(y)] = [\phi_m^{in}(x), \phi_m^{in}(y)]. \tag{19}$$

$$x_0, y_0 > T$$

In our case, since $G_{Rm}(x)$ is not zero, but exponentially damped for $x_0 < 0$, eq. (19) is not true. However, we will still have, that all contributions to the right hand side of eq. (18), except the first one, vanish exponentially in the limit of large times, and therefore, to every order of perturbation theory,

$$[\phi(x), \phi(y)] = [\phi_m^{out}(x), \phi_m^{out}(y)] = [\phi_m^{in}(x), \phi_m^{in}(y)], \tag{20}$$

again to be understood in the L.S.Z. sense.

The last equality of (20) ensures the unitarity of the S-matrix, provided an out vacuum can be found, implying a restriction on the spatial range of the potential, (to avoid catastrophic pair creation) just as in the real mass case [8].

From (13) and (12) one easily obtains the "out field" in terms of the "in field", and computes scattering amplitudes,

$$\langle k'\text{out}|k\text{in}\rangle = \langle 0\text{out}|0\text{in}\rangle \left(\delta(k'-k) - \frac{i}{(2\pi)^3 2\sqrt{\omega_k\omega_{k'}}}\tilde{V}(k'-k)\right.$$

$$\left. + \frac{i}{(2\pi)^7 2\sqrt{\omega_k\omega_{k'}}}\int d^4k'' \frac{\tilde{V}(k'-k'')|M^2-m^2|^2\tilde{V}(k''-k)}{(k''^2-m^2+i\epsilon)(k''^2-M^2)(k''^2-M^{*2})} + \ldots\right) \tag{21}$$

with

$$\omega(k) = \sqrt{k^2+m^2}\,,$$

$$V(x) = \frac{1}{(2\pi)^4}\int \tilde{V}(k)\,e^{-ikx}d^4k\,,$$

and the Feynman prescription $i\epsilon$ applied only to the real mass (contour C_{Fm} in fig. 1).

One has therefore, obtained a set of modified "Feynman rules" for a unitary theory with complex masses, in the case of interaction with an external potential. For the sake of illustration we present an explicit computation of the amplitudes and a direct check of unitarity up to 2nd order perturbation theory in appendix A.

With the additional technical assumption that the potential has compact support in time and is a bounded function, it is possible to prove unitarity of the theory based on eq. (10), independent of perturbation theory (or rather showing its convergence), using methods closely related to the ones employed in ref. [8]. Since the proof clarifies the interplay between unitarity and non-causality it will be presented, in appendix B.

It is appropriate to end this section be remarking that there is no canonical formalism underlying our modified Yang-Feldman eq. (10). This becomes clear by noticing that, for time-dependent potentials, the equal time commutation relations depend explicitly on time through the potential. In this case, the field algebras at different times are not equivalent, but for potentials that are slowly varying within the acausal interval, one can still speak of an approximate equivalence. In the static limit, one has, a posteriori, due to the time invariance of the Wightman functions, a time translation generator. In the general case however, energy has only an asymptotic meaning, being defined only for the incoming and outgoing states.

In a certain sense, the theory thus developed, is intermediate between a pure S-matrix and a local quantum theory, since meaningful observations will have to be done in space-time regions large compared to the critical acausal length $1/\mathrm{Im}M$.

It should also be stressed that our procedure requires complex masses. Real masses will lead to a much more violent acausality, falling off as power, and also to unitarity troubles.

The modifications leading to our eq. (10) amount to having the complex masses only as a parametrization of an acausal propagation law for physical particles. Since, in the present treatment, the complex masses have no particle manifestation whatsoever, the indefinite metric of the usual complex mass theories [1–5, 12], plays no role here.

3. FULLY QUANTIZED THEORY

We shall proceed now to a discussion of the fully quantized theory, taking for the sake of illustration the simplest case $J(x) = \lambda : \phi^2(x) :$.

$$\frac{(\Box + m^2)(\Box + M^2)(\Box + M^{*2})}{|M^2 - m^2|^2} \phi(x) = \lambda : \phi^2(x) : + \delta m^2 \frac{(\Box + M^2)(\Box + M^{*2})}{|M^2 - m^2|^2} \phi , \tag{22}$$

where the (finite) mass renormalization counter-term has been explicity introduced.

We shall solve (22) by iterations of the modified Yang-Feldman equation (10). Up to 2nd order, performing the Wick-contraction, we obtain,

$$\begin{aligned} \phi(x) &= \phi_m^{\text{in}}(x) + \lambda \int G_{\text{R}m}(x-y) : \phi_m^{\text{in}2}(y) : \mathrm{d}^4 y \\ &+ 2\lambda^2 \int G_{\text{R}m}(x-y)\, G_{\text{R}m}(y-z) : \phi_m^{\text{in}2}(z)\, \phi_m^{\text{in}}(y) : \mathrm{d}^4 y\, \mathrm{d}^4 z \qquad (23) \\ &+ \int G_{\text{R}m}(x-y)\, \mathrm{d}^4 y [2\lambda^2 \\ &\times \int G_{\text{R}m}(y-z)\, \Delta^1(y-z)\, \mathrm{d}^4 z\ \phi_m^{\text{in}}(z) + \delta m^2\, \phi_m^{\text{in}}(y)] \\ &+ \text{higher order} + \ldots \end{aligned}$$

with

$$\Delta^1(y-z) = i\, [\phi_m^{\text{in}}(y), \phi_m^{\text{in}}(z)]_+ . \tag{24}$$

From (23), the mass renormalization is given to second order by

$$\delta m^2 = F(m^2) , \tag{25}$$

with

$$F(p^2) = \frac{2\lambda^2}{(2\pi)^3} \int \frac{\mathrm{d}^4 k\, \delta(k^2 - m^2) |M^2 - m^2|^2}{\{(p-k)^2 - m^2\}\{(p-k)^2 - M^2\}\{(p-k)^2 - M^{*2}\}} \tag{25}$$

Since the self-energy integral (25), will, in higher orders, also contribute to the radiative corrections to scattering, and is the relevant quantity for calculating the response of the field to an external perturbation, we are interested in its behaviour for arbitrary p^2. The asymetry between the contraction function and the Green's function, comes from our requirement that the complex masses should have no asymptotic manifestations, and will play an important role in the analytic structure of $F(p^2)$. It will lead, on one hand, to unitarity, (no pair of complex conjugate particles exists in our approach), and, on the other hand, to a much more serious acausality than in potential scattering.

It is readily seen that, after performing an integration on a time-like direction, (25) (25) becomes an absolutely convergent integral in the remaining variables provided $0 < p^2 < 4\,m^2$. Furthermore, the result of this integration is independent of the particular time-like direction one started with, assuring thus, the Lorentz invariance of the integral for that range of p^2. For $p^2 > 4\,m^2$ one employs the usual retarded prescription $p_0 \to p_0 + i\,\epsilon$ leading also to an unambiguous Lorentz invariant result.

For $-p^2 < p^2 < 0$ the integral (25) diverges. One can, however, define a finite self-energy for all values of p^2, by a procedure of analytic continuation, corresponding to what appears as a natural definition of the still singular product $G_{\mathrm{R}m}(x)\, \Delta^1(x)$, whose Fourier transform is $F(p^2)$ (cf. appendix C). In practice, our procedure amounts to calculate the integral in the center of mass frame, and at the end, substitute p for p_0 keeping the usual retarded prescription.

We thus obtain for $F(p^2)$, the following integral representation, which suffices for a discussion of its analytic structure,

$$F(p^2) = \frac{\lambda^2}{8\pi^2}\left[\sum_{i=1}^{3} C_i \int_{(m+m_i)^2}^{\infty} \frac{\rho(s, m, m_i)\,\mathrm{d}s}{p^2 - s} + \sum_{i=2}^{3} C_i \int_{\epsilon\to 0}^{(m_i - m)^2} \frac{\rho(s, m, m_i)\,\mathrm{d}s}{p^2 - s} \right.$$

$$\left. - \frac{2}{p^2}\,\frac{|M^2 - m^2|^2}{M^2 - M^{*2}} \log \frac{M^2 - m^2}{M^{*2} - m^2}\right] , \tag{26}$$

with

$$\begin{aligned} &m_1 = m, \qquad && m_2 = M, \qquad && m_3 = M^* , \\ &C_1 = 1 , && C_2 = \frac{M^{*2} - m^2}{M^2 - M^{*2}}, && C_3 = C_2^* , \\ &\rho(s, m, m_i) = \sqrt{\frac{(s - (m + m_i)^2)(s - (m - m_i)^2)}{s^2}} . \end{aligned} \tag{27}$$

The analytic structure given by (26) is as follows: $F(p^2)$ is analytic in the p^2 complex plane, with cuts running from $4\,m^2$ to ∞ (unitarity cut), $(m + M)^2$ to ∞, $(m + M^*)^2$ to ∞, $(M - m)^2$ to 0 to $(M^* - m)^2$ and having a pole and a branch point at the origin (Cf. fig. 2).

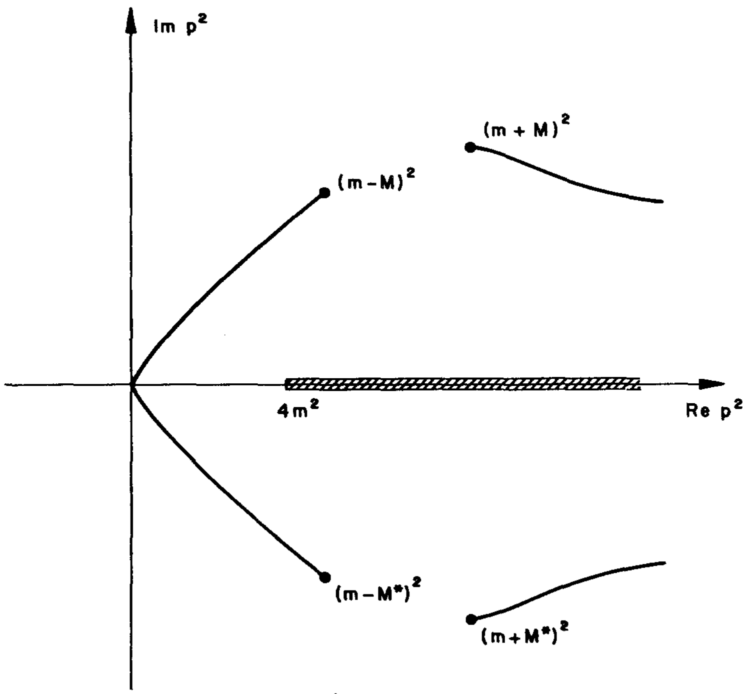

Fig. 2. The cut p^2 plane for the self-energy.

The first three cuts correspond to the ones obtained using the Lee-Wick rules. They are associated intuitively, to "intermediate" states formed of (m, m) , (m, M) and (m, M^*) pairs. There is however no cut corresponding to an (M, M^*) pair, which can be the source of unitarity or Lorentz invariance troubles, and will, if properly treated, lead to a non-analytic singularity [2 - 4].

The extra cut going through the origin, is a new feature coming from the already mentioned asymmetry of our self-energy integral (25), which leads, to an equal footing treatment of the positive and negative energy roots of the complex masses.

This is quite reasonable, since it is through a particle interpretation, which we do not have the complex masses, that the positive root gets singled out.

The pole at the origin, should not be interpreted as indicating a zero mass bound state, or long range forces, since it really corresponds to a zero of the fully corrected propagator

$$G'_{Rm}(p^2) = \frac{|M^2 - m^2|^2}{(p^2 - m_0^2)(p^2 - M^2)(p^2 - M^{*2}) - F(p^2)|M^2 - m^2|^2 + 0(\lambda^4)} \tag{28}$$

$$m_0^2 = m^2 - \delta m^2 \ .$$

From the point of view of the acausal behaviour of the theory, the branch point at the origin is the most important singularity, and is given by (26) as

$$\underset{p^2\sim 0}{F(p^2)} - (\text{pole}) \sim \left[\frac{M^{*2}-m^2}{M^2-M^{*2}}\,\frac{(M^2+m^2)}{(M^2-m^2)}\,\log ip^2 + \text{c.c.}\right]\frac{\lambda^2}{8\pi^2} \tag{29}$$

The connection between the analytic structure of $F(p^2)$ and causality problems [13], becomes particularly clear in our formulation, since (28), is essentially the linear response function to an external perturbation, i.e., the field induced by an external classical source, $J_c(x)$ is, in lowest order in $J_c(x)$,

$$\langle\phi(x)\rangle_{\text{ind}} = -\frac{1}{(2\pi)^4}\int G'_{\text{R}m}(p^2)\,\tilde{J}_c(p)\,\mathrm{e}^{-ipx}\mathrm{d}^4p\ , \tag{30}$$

with

$$\tilde{J}_c(p) = \int J_c(x)\,\mathrm{e}^{ipx}\,\mathrm{d}^4x\ . \tag{31}$$

One can write (28) in the form of a modified Kallen-Lehmann representation

$$G'_{\text{R}m}(p^2) = \int\frac{\sigma(s)\,\mathrm{d}s}{p^2-s} \tag{32}$$

with s in (32) running over the cuts of fig. 2 and σ having δ singularities corresponding to the poles of (28). Considering only the dominating acausal behaviour of (30) which is controlled by the logarithmic singularity at the origin (29), one has with $M^2 = |M^2|\,\mathrm{e}^{i\theta}$, $|M^2|>>m$,

$$\langle\phi(x)\rangle_{\text{ind(acausal)}} \sim \frac{1}{16\,\pi^2\lambda^2}\,\frac{\sin\theta\,\mathrm{e}^{-i\theta}}{i\theta^2|M^2|^2}\int\mathrm{d}^4p\,\mathrm{e}^{-ipx}\int_0^{i\epsilon}\frac{s^2\mathrm{d}s}{p^2-s}\,\tilde{J}_c(p) + \text{c.c.}\ , \tag{33}$$

where the cut in the neighbourhood of the origin has been deformed to lie on the imaginary axis.

Using the asymptotic expansion,

$$\frac{1}{(2\pi)^4}\int\frac{\mathrm{e}^{-ipx}}{p^2-i\delta}\,\mathrm{d}^4p \quad\to -\ \frac{\sqrt{2}}{8(\pi)^{\frac{3}{2}}}(i\delta)^{\frac{1}{4}}\,\frac{\mathrm{e}^{(i\delta)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}}}{(x^2)^{\frac{3}{4}}} \tag{34}$$

$$\delta > 0,\quad x_0 \to -\infty,\quad x^2 > 0\,,$$

we get the following estimate for $J_c(x)$ finitely extended in space-time,

$$\langle\phi(x)\rangle_{\text{ind}} \underset{\substack{x_0\to-\infty \\ x^2>0}}{\to} \sqrt{\tfrac{1}{2}\pi}\,\frac{\Gamma(\frac{13}{2})\sin\theta\,\cos(\theta+\frac{1}{4}\pi)\,\tilde{J}_c(0)}{(x^2)^4\,\lambda^2|M^2|^2\,\theta^2} \tag{35}$$

4. CONCLUSIONS

From the preceding sections it is seen that, although the theory is quite satisfactory in the case of interaction with an external potential, being unitary, and having an exponentially damped acausal behaviour, for the fully quantized theory the situation leaves much to be desired. Although no unitarity troubles appear to the order considered, and one can expect, due to the absence of complex conjugate pairs, that unitarity will be satisfied to all orders, the acausality is no longer exponentially damped but falls off as a power (35).

The reason for this "acausality enhancement" in a fully quantized theory is quite clear: In a relativistic theory the basic acausality of the Green's function $G_{\mathrm{R}m}$ is characterized by an invariant interval $(x_0^2 - x^2)^{\frac{1}{2}} \sim 1/\mathrm{Im}M$. The larger the energy of an process, the closer one is to the light cone, and the larger the effective acausality in time. In a potential theory, (as in sect. 2) the high-energy processes are automatically cut-off, so the acausality stays always under control. In a fully quantized theory, on the other hand, the existence of virtual processes with arbitrarily high-energy, as for instance those contributing to the self-energy integral, reduce the exponential acausality damping to a mere power one.

Such slowly decreasing acausal tails, seem to us a serious shortcoming of the theory, even though one can play with the angle θ, to increase their leading power, and with the large mass M, to make them arbitrarily small.

Besides their serious influence on the acausality, the singularities near the origin of $F(p^2)$, have the effect, that even for very large regulator masses, the low energy behaviour of the theory will be quite different from the usual renormalized theory with no regulators.

These defects lead us to view with some pessimism the physical usefulness of field theories (and by this we mean theories with a given equation of motion and a detailed space-time description) with complex masses. (See however [4].)

On the other hand, it is always possible, to follow the Lee-Wick point of view, and look only at the S-matrix, defined by their modified Feynman rules. If this is done, causality problems have to be discussed in an indirect way, through the behaviour of phase shifts near resonance, and deviations from it appear only for high energy processes [1]. The low energy behaviour of their theory will, for large regulator masses, essentially coincide with the usual one.

The price to pay for this is, that besides losing the direct connection with a simple equation of motion, one loses the space-time description and the possibility of local measurements. And it is, perhaps appropriate to stress, that only in terms of a given equation of motion, that the problem of computing mass shifts due to radiative corrections, (which provided one of the main motivations of the Lee-Wick theory), can be given an unambiguous meaning.

APPENDIX A

In this appendix, we shall illustrate the calculation of the scattering amplitudes, and the direct verification of unitarity, for the interaction with an external potential, starting from the perturbative solution of the modified Yang-Feldman eq.(10). This will be done explicitly up to second order perturbation theory, and can be easily generalized to arbitrary order.

From the iterative solution of (10) one obtains

$$\phi(x) = \phi^{\text{in}}(x) + \int G_{\text{R}m}(x-y)\, K(y, z)\, \phi^{\text{in}}(z)\, \mathrm{d}^4 y\, \mathrm{d}^4 z\ , \tag{A.1}$$

where the kernel K is given by,

$$\begin{aligned} K(y, z) = {} & V(y)\delta^4(y-z) + V(y)\, G_{\text{R}m}(y-z)\, V(z) \\ & + V(y) \int \mathrm{d}^4 y' G_{\text{R}m}(y-y')\, V(y') G_{\text{R}m}(y'-z)\, V(z) + \dots \end{aligned} \tag{A.2}$$

Using the asymptotic condition (12), and remarking that asymptotically only the real pole of the propagator $G_{\text{R}m}$ gives a non-vanishing contribution, one has

$$\phi^{\text{out}}(x) = \phi^{\text{in}}(x) + \int \Delta(x-y)\, K(y, z)\, \phi^{\text{in}}(z) \mathrm{d}^4 y \mathrm{d}^4 z \tag{A.3}$$

with Δ given by (16).

From (A.3) one gets the connection between the incoming and outgoing creation and anihilation operators

$$a^{\text{out}}(k) = a^{in}(k) + \int \theta(\omega_k, k, \omega_{k'} k') a^{\text{in}}(k') \mathrm{d}^3 k' + \int \theta(\omega_k, k, -\omega_{k'} -k') a^{+}(k') \mathrm{d}^3 k' \tag{A.4}$$

with

$$\omega_k = \sqrt{k^2 + m^2}$$

and $\theta(k, k')$ given to 2nd order by

$$\theta(k, k') = \frac{-i}{2(2\pi)^3 \sqrt{|k_0 k_0'|}} \{\widetilde{V}(k'-k) + \int \mathrm{d}^4 k'' \widetilde{V}(k'-k'') \widetilde{G}_{\text{R}m}(k'') \widetilde{V}(k''-k) + \dots\} \tag{A.5}$$

with

$$V(x) = \int \widetilde{V}(k) \mathrm{e}^{-ikx} \mathrm{d}^4 k\ , \tag{A.6}$$

and

$$\widetilde{G}_{\text{R}m}(k) = -\frac{1}{(2\pi)^4} \frac{|M^2 - m^2|^2}{(k^2 - m^2)_{\text{R}} (k^2 - M^2)(k^2 - M^{*2})} \tag{A.7}$$

From (A. 4,5) the scattering amplitude up to 2nd order perturbation is

$$\langle k'\mathrm{out}|k\mathrm{in}\rangle = \langle 0\mathrm{out}|a^{\mathrm{out}}(k')a^{+\mathrm{in}}(k)|0\mathrm{in}\rangle = \delta\,(k'-k)\,\langle 0\mathrm{out}|0\mathrm{in}\rangle - \frac{i\,\widetilde{V}(k'-k)}{2(2\pi)^3\sqrt{\omega_{k'}\omega_k}}$$

$$-\frac{i}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{k'}}}\int \mathrm{d}^3k''\,\frac{\widetilde{V}(k'+k)}{\sqrt{2\omega_{k''}}}\langle 0\mathrm{out}|a^{+\mathrm{in}}(k'')a^{+\mathrm{in}}(k)|0\mathrm{in}\rangle$$

$$-\frac{i}{2(2\pi)^3}\frac{1}{\sqrt{\omega_k\omega_{k'}}}\int \widetilde{V}(k'-k'')\widetilde{G}_{\mathrm{R}m}(k'')\widetilde{V}(k''-k)\mathrm{d}^4k'' + \text{higher order} + \ldots \tag{A.8}$$

The vacuum amplitudes can be obtained from

$$a^{\mathrm{out}}(k)|0\mathrm{out}\rangle = 0 \tag{A.9}$$

which leads to

$$|0\mathrm{out}\rangle = (1 - \frac{1}{8(2\pi)^6}\int \frac{\mathrm{d}^3k\mathrm{d}^3k'}{\omega_k\omega_{k'}}\,|\,\widetilde{V}((k+k'))|^2)|0\mathrm{in}\rangle$$

$$+\frac{i}{4(2\pi)^3}\int \frac{\widetilde{V}(k+k')}{\sqrt{\omega_k\omega_{k'}}}\;a^{+\mathrm{in}}(k)a^{+\mathrm{in}}(k')\mathrm{d}^3k\mathrm{d}^3k'|0\mathrm{in}\rangle$$

$$+\text{ higher order}\ldots \tag{A.10}$$

The amplitude $\langle 0\mathrm{out}|a^{+\mathrm{in}}(k'')a^{+\mathrm{in}}(k)|0\mathrm{in}\rangle$ appearing in (A.8) can now be taken from (A.10) leading to

$$\langle k'\mathrm{out}|k\mathrm{in}\rangle = \langle 0\mathrm{out}|0\mathrm{in}\rangle\,\{\delta\,(k'-k) - \frac{i}{2(2\pi)^3\sqrt{\omega_k\omega_{k'}}}\,(\widetilde{V}(k'-k)$$

$$+\frac{i}{(2\pi)^3}\int\frac{\mathrm{d}^3k''}{2\omega_{k''}}\widetilde{V}(k'+k'')\widetilde{V}(-k''-k) + \int \mathrm{d}^4k''\,\widetilde{V}\,(k'-k'')\,\widetilde{G}_{\mathrm{R}m}(k'')\,\widetilde{V}(k''-k)$$

$$+\text{ higher orders} + \ldots)\} \tag{A.11}$$

The last two terms of (A.11) can be combined by changing the retarded prescription into a Feynman-like one, giving us eq. (22),

$$\langle k'\mathrm{out}|k\mathrm{in}\rangle = \langle 0\mathrm{out}|0\mathrm{in}\rangle\,\{\,\delta\,(k'-k) - \frac{i}{2(2\pi)^3\sqrt{\omega_k\omega_{k'}}}\,(\widetilde{V}(k'-k)\;\;+$$

$$+\int d^4k''\widetilde{V}(k'-k'')\widetilde{G}_{Fm}(k'')\widetilde{V}(k''-k) + \text{higher orders} + \ldots)\} \tag{A.12}$$

with

$$\widetilde{G}_{Fm}(k) = \frac{-|M^2-m^2|^2}{\{k^2-m^2+i\epsilon\}\{k^2-M^2\}\{k^2-M^{*2}\}(2\pi)^4}.$$

One can also compute the production amplitude

$$\langle k_1k_2k_3\text{out}|k\text{in}\rangle = i\sum_{\substack{i=1,2,3\\ i\neq j\neq l}} \frac{\delta(k-k_i)}{\sqrt{3!\,4\,\omega_{k_j}\omega_{k_l}}}\widetilde{V}(k_j+k_l) + \text{higher orders} + \ldots \tag{A.13}$$

and with (A.12), (A.13) directly check unitarity up to second order. The two contributions coming from the imaginary part of the second order term of (A.12), correspond to intermediate states with one resp. three particles, given by the first order terms of (A.12) resp. (A.13).

Up to second order, the imaginary part of the amplitude (A.12), does not depend on the complex masses, being equal to the one given by a normal theory with a Klein-Gordon equation.

The real part, however, depends on the complex masses corresponding to an analytic structure, different from the one obtained in normal potential theory. In higher orders, also the imaginary part, will of course, depend on the complex masses.

APPENDIX B

We show here, that eq. (10), leads in the case of an external bounded potential of compact support in time, to an unitary theory.

It is convenient to introduce auxiliary fields

$$\phi_m(x) = \frac{(\Box-M^2)(\Box-M^{*2})}{(M^2-m^2)(M^{\times 2}-m^2)}\phi(x), \tag{B.1}$$

$$\phi_M(x) = \sqrt{\frac{M^2-M^{*2}}{M^{*2}-m^2}}\frac{(\Box-m^2)(\Box-M^{*2})}{(M^2-m^2)(M^2-M^{*2})}\phi(x), \tag{B.2}$$

$$\phi_{M^*}(x) = (\phi_M(x))^*, \tag{B.3}$$

so that

$$\phi(x) = \phi_m(x) + \sqrt{\frac{M^{*2}-m^2}{M^2-M^{*2}}}\phi_M(x) + \sqrt{\frac{M^2-m^2}{M^{*2}-M^2}}\phi_{M^*}(x). \tag{B.4}$$

Introducing further

$$\alpha_{mi}(x) = \frac{1}{\sqrt{2}} \{\sqrt[4]{-\Delta+m^2}\ \phi_{mi}(x) + \frac{i}{\sqrt[4]{-\Delta+m_i^2}}\ \dot{\phi}_{mi}(x)\}\ , \tag{B.5}$$

$$\beta_{mi}(x) = \frac{1}{\sqrt{2}} \{\sqrt[4]{-\Delta+m_i^2}\ \phi_{m_i}(x) - \frac{i}{\sqrt[4]{-\Delta+m_i^2}}\ \dot{\phi}_{mi}(x)\}, \tag{B.6}$$

with $m_i = m, M, M^*$ and

$$\psi(x) = \begin{pmatrix} \alpha_m(x) \\ \beta_m(x) \\ \alpha_M(x) \\ \beta_M(x) \\ \alpha_{M^*}(x) \\ \beta_{M^*}(x) \end{pmatrix} \tag{B.7}$$

one can rewrite eq. (1) in the form of a six dimensional wave equation

$$i\frac{\partial\psi}{\partial t} = H_0\psi + v(x)\psi\ , \tag{B.8}$$

with

$$H_0 = \begin{pmatrix} \sqrt{-\Delta+m^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{-\Delta+m^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{-\Delta+M^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{-\Delta+M^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-\Delta+M^{*2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{-\Delta+M^{*2}} \end{pmatrix} \tag{B.9}$$

and $v(x)$ a bounded matrix operator whose matrix elements are proportional to

$$\frac{1}{\sqrt{-\Delta+m_i^2}}\ V(x) \frac{1}{\sqrt{-\Delta+m_j^2}}$$

and vanishing for $|t|>T$.

From eq. (10) it is clear that the state space of our theory is the Fock-Space of incoming particles with mass m. We therefore expand

$$\psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}^3k a^{\mathrm{in}}(k)\, \chi_k^+(x) + a^{+\mathrm{in}}(k) \chi_k^-(x) \tag{B.10}$$

with χ^+, χ^- c-number solutions of eqs. (B.8).

The boundary condition (11) requires that for times $t < -T$

$$\chi_k^+(x) = \begin{pmatrix} \mathrm{e}^{-ik\cdot x} \\ 0 \\ \chi_3^+(x) \\ 0 \\ 0 \\ \chi_6^+(x) \end{pmatrix} , \qquad \chi_k^-(x) = \begin{pmatrix} 0 \\ \mathrm{e}^{+ik\cdot x} \\ 0 \\ \chi_3^-(x) \\ 0 \\ 0 \\ \chi_6^-(x) \end{pmatrix} , \tag{B.11}$$

where $k_0 = \sqrt{k^2 + m^2}$ and $\chi_{3,6}^{\pm}$ indicates an as yet indetermined solution of the free equation. Those solutions will be determined by requiring the assymptotic condition (12) to hold.

Assuming for a moment the existence of the classical evolution operator associated with the eq. (B.8) between $-T$ and T we obtain

$$\chi_k^+(x,T) = U\chi_k^+(x, -T) , \qquad \chi_k^-(x,T) = U\chi_k^-(x, -T) . \tag{B.12}$$

Trough the imposition of the asymptotic condition (12),

$$\begin{aligned} U_{33}\chi_3^+(x) + U_{36}\chi_6^+(x) &= -U_{31}(\mathrm{e}^{-ikx}) , \\ U_{63}\chi_3^+(x) + U_{66}\chi_6^+(x) &= -U_{61}(\mathrm{e}^{-ikx}) , \end{aligned} \tag{B.13}$$

$$\begin{aligned} U_{33}\chi_3^-(x) + U_{33}\chi_6^-(x) &= -U_{32}(\mathrm{e}^{+ikx}) , \\ U_{63}\chi_3^-(x) + U_{66}\chi_6^-(x) &= -U_{62}(\mathrm{e}^{+ikx}) . \end{aligned} \tag{B.14}$$

Eqs. (B.13, 14) can be inverted, at least for sufficiently weak potentials, giving thus a complete determination of $\chi^{\pm}(x)$ for $t < -T$. It is clear that the acausal features of the theory are introduced precisely through eqs. (B.13, 14) which adjust the initial conditions depending on the future values of the potential.

To show the existence of the time evolution operator is suffices to prove that Dyson's expansion is convergent in norm.

This is the case since:

(a) Introducing in the usual fashion the norm of an operator as

$$||A|| = \max \frac{(\chi A^+, A\chi)}{(\chi, \chi)} , \tag{B.15}$$

with

$$(\chi,\chi) = \int d^3x \sum_{i=1}^{6} \chi_i^*(x)\,\chi_i(x) \tag{B.16}$$

the operator

$$H_{\text{int}}(t) = e^{itH_o}\, v(t)\, e^{-itH_o} \tag{B.17}$$

is bounded in norm as long as $V(x)$ is a bounded function.

(b) The time evolution operator

$$U = e^{-iH_o 2T}\, T\ \exp -i \int_{-T}^{T} H_{\text{int}}(t')\,dt' \tag{B.18}$$

exists and is defined by Dyson's expansion.

Furthermore, it is easily seen that U is pseudo-unitary with respect to an indefinite metric given by

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} . \tag{B.19}$$

With (B.10, 12) one can relate the outgoing operators,

$$\psi^{\text{out}}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k\, \{a^{\text{out}}(k)\, \xi_k^+(x) + a^{+\text{out}}(k)\xi_k^-(x)\} , \tag{B.20}$$

where

$$\xi_k^+(x) = \begin{pmatrix} e^{-ikx} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} , \qquad \xi_k^- = \begin{pmatrix} 0 \\ e^{ikx} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} , \tag{B.21}$$

to the incoming ones, through

$$a^{\text{out}}(k) = \int d^3k'\, (\xi_k^+, \eta U \chi_{k'}^+)\, a^{\text{in}}(k') + (\xi_k^+, \eta U \chi_{k'}^-)\, a^{+\text{in}}(k') \tag{B.22}$$

where ξ_k^+ is given by (B.21) with $t{=}T$ *and* $\chi_k^\pm$ given by (B.11) with $t{=}{-T}$.

The pseudo-unitarity of U together with (B.11, 13, 14) ensures that the outgoing operators satisfy the correct commutation relations

$$[a^{\mathrm{out}}(k'), a^{+\mathrm{out}}(k)] = \delta(k - k') . \tag{B.23}$$

Provided an out-vacuum can be found in the state-space of incoming particles, which requires a limitation on the range of the potential, to avoid catastrophic pair creation [8], (B.23) leads to the existence of an unitary S-matrix such that

$$a^{\mathrm{out}}(k) = S^{-1} a^{\mathrm{in}}(k) S ; \qquad |0\mathrm{out}\rangle = S^{-1} |0\mathrm{in}\rangle . \tag{B.24}$$

APPENDIX C

We shall analyse here in greater detail, the self-energy integral (25).
With

$$p = (p_0\; 0\; 0\; p_1)$$

introducing cylindrical coordinates

$$k_0 , \qquad k_1 = k_3 , \qquad l = \sqrt{k_x^2 + k_y^2} , \qquad \theta = \operatorname{arctg} \frac{k_y}{k_x} ,$$

we obtain

$$F(p) = \frac{\lambda^2}{2\pi} \int_0^\infty l\,\mathrm{d}l \sum_{i=1}^{3} C_i [I_1(p, m(l), m_i(l)) + I_2(p, m(l), m_i(l))] \tag{C.1}$$

with

$$I_1(p, m(l), m_i(l)) = \int \tfrac{1}{2}\,\mathrm{d}k_1 \left[\frac{1}{\sqrt{k_1^2 + m^2(l)}} + \frac{1}{\sqrt{(p_1 - k_1)^2 + m_i^2(l)}} \right]$$
$$\times \frac{1}{p_0^2 - (\sqrt{k_1^2 + m(l)^2} + \sqrt{(p_1 - k_1)^2 + m_i^2(l)^2})^2} \tag{C.2}$$

$$I_2(p, m(l), m_i(l)) = \int \tfrac{1}{2}\,\mathrm{d}k_1 \left[\frac{1}{\sqrt{k_1^2 + m^2(l)}} - \frac{1}{\sqrt{(p_1 - k_1)^2 + m_i^2(l)}} \right]$$

$$\times \frac{1}{p_0^2 - (\sqrt{k_1^2 + m(l)^2} - \sqrt{(p_1 - k_1)^2 + m_i^2(l)^2})^2} , \tag{C.3}$$

where

$$m_i(l) = \sqrt{m_i^2 + l^2}$$

and m_i, C_i introduced in (27).

A detailed analysis of the contributions from the I_1 terms has been given by Sudarshan et al. [3] and will not be reproduced here. As a result of their investigation one is immediately led to the first three integrals of eq. (26).

We shall concentrate on the I_2 contributions which will lead to the two remaining integrals of (26) corresponding to the extra-cut going through the origin of fig. 2.

It will become clear in what follows that $I_2(p, m(l), m(l)) = 0$ so we will have to consider only the last two terms of (C.1).

Introducing the variable s,

$$s = (\sqrt{k_1^2 + m^2(l)} - \sqrt{(p_1 - k_1)^2 + m_i^2(l)})^2 - p_1{}^2 = (q^\mu + q'^\mu)(q_\mu + q'_\mu) \quad \text{(C.4)}$$

with

$$\begin{aligned} q^\mu &= (\sqrt{k_1{}^2 + m^2(l)}\,, k_1)\,, \\ q'^\mu &= (-\sqrt{(p_1 - k_1)^2 + m_i(l)^2}\,, (p_1 - k_1))\,, \end{aligned} \quad \text{(C.5)}$$

and recalling that

$$\frac{\mathrm{d}s}{\mathrm{d}k_1} = 2\left[\frac{1}{\sqrt{k_1^2 + m^2(l)}} - \frac{1}{\sqrt{(p_1 - k_1)^2 + m_i^2(l)}}\right]\epsilon^{\mu\nu}\, q_\mu q'_\nu \quad \text{(C.6)}$$

we are led to

$$I_2(p.\ m(l), m_i(l)) = 2\oint \frac{\mathrm{d}s}{\sqrt{\{s - (m(l) - m_i(l))^2\}\{s - (m(l) + m_i(l))^2\}}\,(p^2 - s)} \quad \text{(C.7)}$$

where the integration contour, depicted in fig. 3 for $m_i = M$, has the following properties: Although p_1 dependent, it never cuts the positive real axis, goes always through the origin, cutting the negative real axis for $s < -p_1^2$, and encircles the branch point $(m(l) - m_i(l))^2$ and the respective cut of the square root function.

From the above it is clear that for $p^2 > 0$ the contour can be deformed so that

$$I_2(p, m(l), m_i(l)) = 4\int_0^{(m(l) - m_i(l))^2} \frac{\mathrm{d}s}{\sqrt{\{s - (m(l) - m_i(l))^2\}\{s - (m(l) + m_i(l))^2\}}\,(p^2 - s)} \quad \text{(C.8)}$$

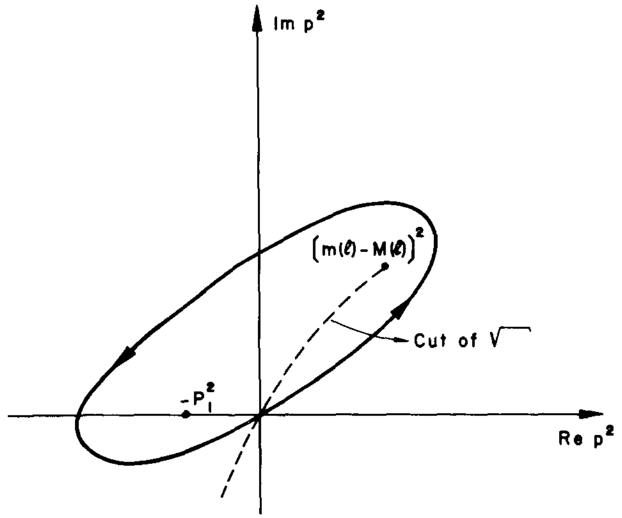

Fig. 3. The p_1 dependent integration contour for $I_2(p, m(l), M(l))$.

and inserting (C.8) into (C.1) one reproduces the representation of eq. (26). (In particular for $m_i = m$ one has $I_2 = m$ one has $I_2 = 0$ as anticipated.) Although separately, each one of the last two integral in (C.1) would diverge at the upper end, their sum leads to a finite result. Careful treament of the upper limits gives rise to the pole of eq. (26).

For $-p_1^2 < p^2 < 0$, in deforming the integration contour of (C.7), as in (C.8), one picks up an extra contribution, since now for all l, p^2 is inside the contour,

$$I_2\,(p, m(l), m_i(l)) = \frac{\pm\, 8\pi i}{\sqrt{\{p^2 - (m(l) - m_i(l))^2\}\,\{\, p^2 - (m(l)) + m_i(l))^2\}}} + R$$

with R given by (C.8).

Inserting now (C.9) into (C.1) one gets from the extra terms a contribution that for large l behaves as

$$\int^{\infty} l\mathrm{d}l \sum_{i=2}^{3} C_i I_2\,(p, m(l), m_i(l)) \sim \frac{(M^{*2} + M^2 - 2m^2)\, 8\pi i}{(M^{*2} - M^2)\sqrt{-p^2}} \int^{\infty} \mathrm{d}l\,. \quad \text{(C.10)}$$

In this case there is no cancellation of divergent integrals and one is left with an infinite result.

This divergence is a manifestation of the still singular nature of the product

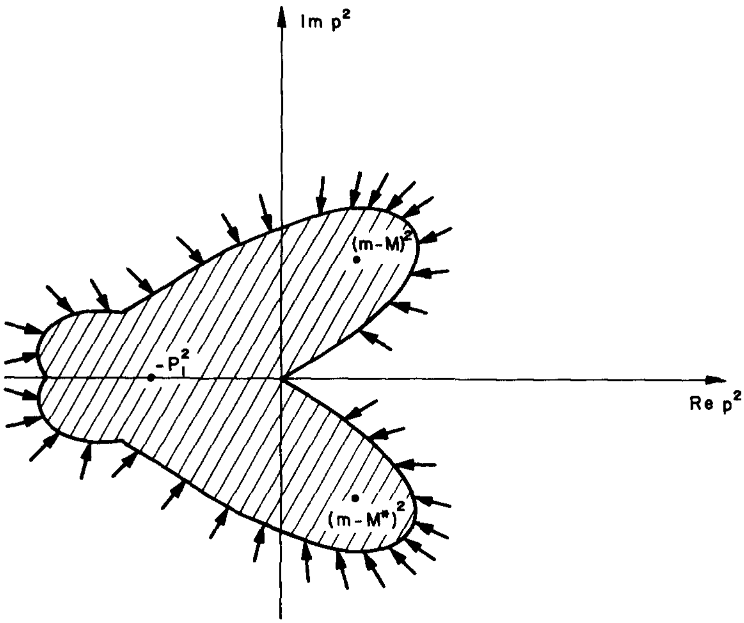

Fig. 4. The self-energy is defined everywhere by analytic continuation into the hatched region.

$G_{Rm}(x)\Delta^1(x)$ whose Fourier transform is the self-energy integral.

For sufficiently negative values of p^2, one can again deform the integration contour of (C.7) without picking up any extra term, and therefore (C.8) is valid once more, leading to the representation of eq. (26).

To give $F(p^2)$ finite values for all p^2, we employ a procedure of analytic continuation, starting from values of p^2 outside the hatched region of fig. 4, were representation (26) holds, and compressing this region, which is not a natural boundary of analyticity, into the cut, going through the origin, of fig. 2.

In this way we enforce representation (26) everywhere and obtain a finite self-energy for all values of p^2.

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